1 Properties of Powers with Integer Exponents

MATERIALS

Calculator

LESSON OVERVIEW

The terms *power*, *base of a power*, and *exponent of a power* are defined. Students write and evaluate expressions with positive integer exponents. They begin with a context using the power with a base of 2. Students then investigate positive and negative integer bases, where the negative sign may or may not be raised to a power depending on the placement of parentheses. Some expressions also contain variables.

ALGEBRA I TEKS

Mathematical Process Standards

(1) The student uses mathematical processes to acquire and demonstrate mathematical understanding.

The student is expected to:

A.1D communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

A.1F analyze mathematical relationships to connect and communicate mathematical ideas.

A.1G display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Number and Algebraic Methods

(11) The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms.

The student is expected to:

A.11B simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.

ELPS

(1) Learning Strategies

The student is expected to:

(E) internalize new basic and academic language by using and reusing it in meaningful ways in speaking and writing activities that build concept and language attainment.

(3) Speaking

The student is expected to:

(D) speak using grade-level content area vocabulary in context to internalize new English words and build academic language proficiency.

ESSENTIAL IDEAS

- Large numbers that have factors that are repeated can be written as a product of powers.
- Placement of parentheses in an expression with an exponent determines what portion of the expression is raised to the exponent.
- When a negative value is raised to an exponent that is an even integer, the simplified expression is a positive value.
- When a negative value is raised to an exponent that is an odd integer, the simplified expression is a negative value.

LESSON STRUCTURE AND PACING: 3 DAYS

DAY 1

ENGAGE

Getting Started: Three Generations 5–10 minutes

ESTABLISH A SITUATION

A chart that represents a puppy's lineage is given. Students analyze a pattern between each generation of the puppy's lineage. This activity is designed to engage students in thinking about repeated multiplication patterns, which will be formalized in the lesson.

DEVELOP

Activity 1.1: Review of Powers and Exponents 15–20 minutes

WORKED EXAMPLE, REAL-WORLD PROBLEM SOLVING

Students complete a table detailing the puppy's lineage back seven generations, continuing work from the previous activity. They express the number of sires and dams for each generation in expanded notation and power notation and then answer related questions.

Activity 1.2: Practice with Powers 10–15 minutes

MATHEMATICAL PROBLEM SOLVING

Students investigate the role of parentheses in expressions containing exponents, including negative integers raised to an even or odd power. They evaluate expressions containing exponents and reverse the process to write expressions containing exponents that represent the product of factors.

DAY 2

Activity 1.3: Multiplying and Dividing Powers 20–25 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

A medium sized eBook contains about one megabyte (MB) of information. One gigabyte (GB) is 1024 megabytes, one megabyte (MB) is 1024 kilobytes, and one kilobyte is 1024 bytes. Students calculate the storage capacity of eBooks and jump drives. This activity provides a context that creates the opportunity for students to perform multiplication and division on expressions with exponents.

Activity 1.4: Product of Powers 15–20 minutes

WORKED EXAMPLE, MATHEMATICAL PROBLEM SOLVING

Students use expanded notation of expressions to develop the product of powers rule. The product of powers rule states that to multiply powers with the same base, keep the base the same and add the exponents.

DAY 3

Activity 1.5: Quotient of Powers 5–10 minutes

MATHEMATICAL PROBLEM SOLVING

Students use expanded notation of expressions to develop the quotient of powers rule. The quotient of powers rule states that to divide powers with the same base, keep the base the same and subtract the exponents.

Activity 1.6: Powers Equal to 1 and Numbers Less Than 1 25–30 minutes

MATHEMATICAL PROBLEM SOLVING

Students use patterns, the product of powers rule, and the quotient of powers rule to develop the meaning of the zero power and negative exponents. They complete symbol manipulation to generate equivalent fractional expressions, expressions with positive exponents, and expressions with negative exponents.

DEMONSTRATE

Talk the Talk: Simplifying 10 minutes

EXIT TICKET APPLICATION

The rules for operating with powers are summarized in a table. Students use these rules to simplify expressions.

Getting Started

ENGAGE

Three Generations

Facilitation Notes

In this lesson, students are given a scenario represented in chart form. Students analyze a pattern between each generation of the puppy's lineage. This activity is designed to engage students in thinking about repeated multiplication patterns, which will be formalized in the lesson.

Have a student read the introduction. As a class, discuss the lineage provided in the chart. Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

DIFFERENTIATION STRATEGY

Access for All

If students look at the sequence of numbers without the context, they might recognize the pattern of repeated addition of the previous number. Acknowledge that this is a correct pattern, but it is not as useful as the pattern of multiplication by 2; it is easier to write and use a pattern where the number stays constant instead of being based upon a previous term.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Express the number of dogs in each generation as a sequence. • Why does this pattern make sense for this context?

STAMP THE LEARNING

Summary

Charts can be used to organize data and look for patterns.

ACTIVITY 1.1 Review of Powers and Exponents

DEVELOP

Facilitation Notes

In this activity, students complete a table detailing the puppy's lineage back seven generations, continuing work from the previous activity. They express the number of sires and dams for each generation in expanded notation and power notation and answer related questions.

Have students work with a partner or in a group to complete Question 1. Note that students are only completing the second column of the table. Have students share their strategies for completing the number of sires and dams in Rickson's lineage. This question does not require students to use powers.

AS STUDENTS WORK, LOOK FOR

The use of powers. Although it is not expected here, some students may have recognized the pattern and started using exponential notation.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What do the values 2, 4, and 8 in the table represent in the context?
Probing	• How did you complete the table without extending the diagram? • Why do the values in this problem increase by a factor of two for each generation?

Ask a student to read the information and definitions following the table aloud. Analyze the Worked Example as a class. Then, have students work with a partner or in a group to complete Questions 2 and 3. Have students share their completed tables with another partner or group. Share responses as a class.

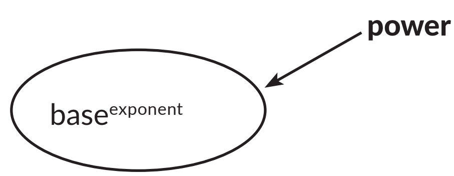
COMMON MISCONCEPTION

Students may inaccurately refer to the exponent as the power. This misconception is strengthened by the fact that 2^7 may be read as "2 to the 7th power." Remind students that a power is comprised of a base and an exponent.

DIFFERENTIATION STRATEGY

Access for All

Have students take notes on the definitions and Worked Example.



- Include a diagram.
- Have students write "= 128" in the Worked Example and then write an additional example to demonstrate that placement of the base and exponent makes a difference: $7^2 = (7)(7) = 49$.
- Have students write examples that include variables, such as 3^x and x^3. After they complete Questions 2 and 3, have them write these powers in expanded notation, $3^x = (3)(3)(3)\ldots x$ times and $x^3 = (x)(x)(x)$. Students cannot solve for a single value because they do not know the value of *x*.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What are the different ways to read each power of 2 in the table?
Probing	• How is the generation number related to the exponent of the power representing the generation?
Seeing structure	• What is the difference between a power and an exponent?

Have students answer Questions 4 and 5 in partners or in a group. Share responses as a class.

AS STUDENTS WORK, LOOK FOR

- Use of expanded notation by typing twelve 2s into the calculator.
- Knowledge and use of the ^ key or a^b key to enter the exponent into a calculator.

COMMON MISCONCEPTION

Students may attempt to answer Question 5, asking for a total, by using exponents exclusively. Discuss the fact that addition is still required to solve this problem.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you calculate 2^{12} using technology? What is another way? • How did you calculate the total number of sires and dams in all three generations?

STAMP THE LEARNING

Summary

A power is an expression used to represent the product of a repeated multiplication. The base of a power is the expression that is used as a factor in the repeated multiplication, and the exponent of a power is the number of times that the base is used as a factor in the repeated multiplication.

ACTIVITY 1.2 Practice with Powers

Facilitation Notes

In this activity, students investigate the role of parentheses in expressions containing exponents, including negative integers raised to an even or odd power. They evaluate expressions containing exponents and reverse the process to write expressions containing exponents that represent the product of factors.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class. Then, answer Questions 3 and 4 as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How do you know how many times to write the negative sign when writing an exponent as a product? • How many negative signs are in the expanded expression for -1^5? $(-1)^5$? • What is the sign of a positive number raised to an odd power? An even power?

Seeing structure	• Why does the base in part (b) include the negative sign, but the base in part (c) does not? • What's the difference in the bases in parts (a) through (d) and parts (e) through (h)?

DIFFERENTIATION STRATEGY

Access for All

- Have students create a table similar to the Talk the Talk table for taking notes on the exponent rules developed throughout this lesson.
- Suggest students write the rules in their own words instead of copying the rules from the book.
- Encourage students to add additional rules or examples, such as patterns of negative bases raised to a power, to their tables.
- As you begin the Talk the Talk, have students compare their tables to the completed table provided.

Power Operations	Examples	Rule
Negative signs inside or outside parentheses	$(-11)^3 = (-11)(-11)(-11)$ $-11^3 = -(11)(11)(11)$	If a negative sign isn't in parentheses with the base, it does not get raised to a power.
Number of negative signs raised to a power	$(-1)^2 = (-1)(-1) = 1$ $(-1)^3 = (-1)(-1)(-1) = -1$	$\text{negative}^{\text{odd \#}} = \text{negative}$ $\text{negative}^{\text{even \#}} = \text{positive}$
Multiplying		
Power of a power		
Dividing		
Zero power		
Negative powers		
Negative powers in the denominator or a fraction		

To close the Day 1 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

A negative number raised to an odd power results in a negative product, and a negative number raised to an even power results in a positive product.

ACTIVITY 1.3

Multiplying and Dividing Powers

Facilitation Notes

In this activity, students calculate the storage capacity of eBooks and jump drives. This activity provides a context that creates the opportunity for students to perform multiplication and division on expressions with exponents.

To begin the Day 2 session, have a student read the Essential Question aloud.

Ask a student to read the introduction and examine the Worked Example as a class. Call attention to the side note that provides the conversion rates for gigabytes, megabytes, kilobytes, and bytes. Have students work with a partner or in a group to complete Question 1. Share responses as a class.

COMMON MISCONCEPTION

Students may be confused by the conversion 1 kilobyte = 1024 bytes because they have learned that the prefix kilo means 1000. Acknowledge their understanding of prefixes and why, rightfully so, they may be confused. Follow-up with the reasoning behind this conversion and refer students to the second Worked Example, which addresses the fact that computers use a base-2 system instead of a base-10 system, then offer this backstory. Originally, computer scientists used the metric system prefixes and chose kilobyte for 1024 bytes because it was about 1000 bytes. In 1998, the International Electrotechnical Commission (IEC) adopted a different set of prefixes for the binary multiples to avoid confusion, such as using kibibytes for 1024 bytes instead of kilobytes; however, the IEC's prefixes have not caught on and are not commonly used.

DIFFERENTIATION STRATEGY

Access for All

Before proceeding with any calculations, ask students if they have any idea why the number 1024 was used for these conversions. They may or may not realize that if they continued multiplying by 2 in the table in Activity 1, they would have reached 1024. Have them go back to the table and include 2^8, 2^9, and 2^{10}. This will assist them in completing Question 2.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What other types of devices use memory?
Probing	• How did you determine whether to multiply or divide in this problem? • Could you solve this division problem another way? • Did you multiply across and then divide, or did you divide out common values before multiplying?

Analyze the Worked Example before Question 2 as a class. Have students work with a partner or in a group to complete Questions 2 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• What is a quotient?
Probing	• For Question 2, can you justify your responses using expanded notation? • Are all of the bases the same value? • Is the exponent in each product the sum of the exponents of the factors? • Is the exponent in the quotient the difference of the exponents in the numerator and the exponents in the denominator?

DIFFERENTIATION STRATEGY

Access for All

- Students may not be convinced that their exponential answers in Question 2 are equivalent to their answers from Question 1. Have them enter their answers from Question 2 in the calculator to verify that they match those from Question 1.
- Students may ask if they could have divided out 2^8 from the numerator and denominator in Question 2, part (c). This is a correct alternative strategy.

Summary

STAMP THE LEARNING

When multiplying powers with the same base, keep the base the same and add the exponents. When dividing powers with the same base, keep the base the same and subtract the exponents.

ACTIVITY 1.4 Product of Powers

Facilitation Notes

In this activity, students use expanded notation of expressions to develop the product of powers rule and the power of a power rule.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• Which property states that it doesn't make a difference how you order the factors?

Probing	• Why are some of your responses expressed as a product of powers rather than a single power? • How does this relationship apply to parts (c) and (d), where not all original expression factors have an exponent? • How does your rule address both the base and exponent of the resulting power? • How could you modify your rule for cases when the factors have different bases?
Seeing structure	• What is another way to describe this relationship?

Analyze the Worked Example as a class. Discuss how $(4^2)^3$ is rewritten as a product of 4s. Have students work with a partner or in a group to complete Questions 6 through 8. As a class, have students share the rules they generated in Question 8.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Explain why you addressed the negative signs differently in parts 6 (c) and 6 (d). • Do you think you will get the same result for $(8^3)^2$? Explain your thinking.
Seeing structure	• What is another way to describe this relationship? • How does your rule address both the base and exponent of the resulting power?

Have students use their rules to answer Questions 9 through 11 with a partner. Share responses as a class.

AS STUDENTS WORK, LOOK FOR ERRORS SUCH AS

- Multiplication of the bases and/or exponents
- Failure to consider bases without an exponent, rather than using an exponent of 1 in these cases.

DIFFERENTIATION STRATEGIES

Just in Time Support

- Suggest that students continue using expanded notation until they are comfortable with the rules.
- While most exponents in this lesson are small enough to use expanded notation, model for students how to deal with larger exponents when they arise. For example, for $2^{50} \cdot 2^{35}$, write a parallel problem using smaller numbers to recreate the rule. Since $2^3 \cdot 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$, that must mean that I keep the base the same and add the exponents.
  Therefore, $2^{50} \cdot 2^{35} = 2^{85}$.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• Which rule did you apply to rewrite this expression? • In parts (d) and (e), how do you know the base for the given exponent? • In part (f), why is your response expressed as a single power rather than a product with bases 2 and 3? • What would you say to Ramon to address the error in his thinking and correct his work?
Reflecting and justifying	• Use expanded notation to verify your response.
Seeing structure	• Why don't any of the rules we created apply to this problem?

To close the Day 2 session, have students reread the Essential Question and read the activity summaries to the class.

Summary

STAMP THE LEARNING

When multiplying powers with the same base, keep the base the same and add the exponents. When determining a power of a power, keep the base the same and multiply the exponents.

ACTIVITY 1.5

Quotient of Powers

Facilitation Notes

In this activity, students use expanded notation to develop the quotient of powers rule.

To begin the Day 3 session, have a student read the Essential Question aloud.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• What is another way to describe this relationship? • Why do dividing out common factors and subtracting exponents have the same result?
Probing	• How does your rule address the bases in the original problem? • How does your rule address both the base and exponent of the resulting power?

DIFFERENTIATION STRATEGY

Access for All

As in Activity 1.3, students may ask if they could have divided out 9s from the numerator and denominator. Along with having students demonstrate this correct alternative strategy for the class, have them explain why dividing out common factors and subtracting exponents are equivalent operations.

Have students work individually to answer Question 4 before sharing and correcting, if necessary, their answers with a partner or in groups. Share results as a class.

QUESTIONS TO SUPPORT DISCOURSE

Seeing structure	• Why can't you apply your rule for the quotient of powers to solve this problem? • Write and solve another problem for which you can apply your rule for the quotient of powers.

STAMP THE LEARNING

Summary

When dividing powers with the same base, keep the base the same and subtract the exponents.

ACTIVITY 1.6 Powers Equal to 1 and Numbers Less Than 1

Facilitation Notes

In this activity, students use patterns, the product of powers rule, and the quotient of powers rule to develop the meaning of the zero power and negative exponents.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How can you rewrite 4 as a power? What is another way? • Why is the exponent zero in each case? • Using the original fractions' values, what must each of these expressions with an exponent of zero also equal? • What is the value of 4^0? Why? • What is the value of 0^4? Why?

DIFFERENTIATION STRATEGY

Access for All

It may be difficult for students to make sense of a number with an exponent of 0. Revisit the dog lineage in the Getting Started; label Generation 0 and see that it has 1 dog, Rickson.

As a class, complete Questions 4 and 5. Discuss the patterns in the exponents that students notice as they multiply and divide by 10.

QUESTIONS TO SUPPORT DISCOURSE

Reflecting and justifying	• Why does the expression's value increase when you increase the exponent and have a whole-number base?
Probing	• Why is each numerator expressed as 100 when you rewrite the fraction using the definition of powers? Why is the base 10? Why is the exponent 0? • Why is each result using the quotient of powers rule a negative number? • Following the same pattern, how would you write $\frac{1}{1{,}000{,}000}$ with a negative exponent?
Seeing structure	• Select a row and read it horizontally to explain the equivalencies. • How is an expression with a negative exponent related to its fractional representation without exponents?

DIFFERENTIATION STRATEGIES

Access for All

Materials Needed: Graphing Technology

- Have students complete a table in the margin to show the continuum from positive to negative exponents.
- Have students make the connection between a positive exponent and the number of zeroes in the number and a negative exponent and the number of places behind the decimal point (not the number of zeros).
- Have students enter $y = 10^x$ in the graphing technology and pull up a table to verify their results. Note that students may see scientific notation in the table beyond the exponent values of 3 and -3.

Power	Fractional Form	Decimal Form
10^3	1000	1000
10^2	100	100
10^1	10	10
10^0	1	1
10^{-1}	$\frac{1}{10}$	0.1
10^{-2}	$\frac{1}{100}$	0.01
10^{-3}	$\frac{1}{1000}$	0.001

Have students use the patterns they recognized in Questions 4 and 5 to complete Question 6 with a partner or in a group. Share results as a class.

DIFFERENTIATION STRATEGY

Just in Time Support

Students may benefit by including an intermediate step when moving from a fraction to a negative exponent that is not a base of 10. For example, $\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• What expression did you use to rewrite the numerator of 1? Why did you use a base of 2? An exponent of 0? • Explain your steps to rewrite one of the fractions as a power.
Seeing structure	• What patterns do you notice between the given values and the expressions as powers?

Have students work in pairs or groups to complete Questions 7 through 12. Share responses as a class.

DIFFERENTIATION STRATEGIES

Just in Time Support

When evaluating complex fractions, suggest avoiding the complex fraction and calculating with the negative exponents instead. For example, rewrite $\frac{1}{3^{-2}}$ as $\frac{3^0}{3^{-2}}$ (similar to Question 5) and then subtract exponents, so $3^{0-(-2)} = 3^2$.

Challenge Opportunity

Ask students to write their rules using variables for the base and exponents. If they need a hint, suggest that they constrain the exponent variable as a positive value.

QUESTIONS TO SUPPORT DISCOURSE

Probing	• How did you determine the decimal equivalent? • How could you rewrite the given expression as a fraction without exponents? How could that help you to determine the decimal equivalent? • How did you rewrite the expression with a positive exponent as an expression with a negative exponent? • Describe the strategy you used to rewrite the expressions that had a negative exponent in the denominator. • What is another way to state this rule?

Seeing structure	• What pattern do you notice between the expressions with positive and negative exponents? • Compare the given expressions in the last three rows of the table to those in Question 9. How were your strategies to rewrite the expressions related?

Summary

STAMP THE LEARNING

The zero power of any number, except 0, is 1. When 1 is in the numerator of a fraction and a power with a negative exponent in is the denominator, it can be rewritten as the power with its opposite exponent. When 1 is in the denominator of a fraction and a power with a negative exponent in is the numerator, it can be rewritten as 1 divided by the power with its opposite exponent in the denominator.

DEMONSTRATE

Talk the Talk

SIMPLIFYING

In this activity, the rules for operating with powers are summarized in a table. Students use these rules to simplify several expressions.

Have students review the table and discuss with a partner or in a group.

DIFFERENTIATION STRATEGIES

Access for All

- If each student has completed a notes template throughout the lesson, have them compare their notes with the table provided.

Materials Needed: Index Cards

- Make flashcards with the algebraic versions of the rules, removed from all other columns in the table. Provide a card to each group and have the group present the rule, its meaning in words, and an example to the class.

Have students work with a partner or in a group to complete Questions 1 through 20. Share responses as a class.

QUESTIONS TO SUPPORT DISCOURSE

Gathering	• How do the rules demonstrate that the bases must be the same to apply the properties? • What is the base for each exponent?
Seeing structure	• Provide an example to demonstrate this rule. • In the two rules for negative exponents, why do you think you have to constrain the exponent variable to $m > 0$? • Which property of exponents relates to this problem? How can you tell?

Probing	• Which rule did you apply to evaluate this expression? • Show how you applied the rule to both the variables and numbers. • How did you address the numerical values that are not part of the base?
Reflecting and justifying	• Use expanded notation to show that your solution is correct.

Have students read and answer the Essential Question on the lesson opener page.

STAMP THE LEARNING

Summary

The properties of exponents can be used to simplify numeric and algebraic expressions.

1 Properties of Powers with Integer Exponents

Setting the Stage

- Communicate the objectives and new key terms to look for.
- Tap into your students' prior learning by reading the narrative statement.
- Provide a sense of direction by reading the Essential Question.

OBJECTIVES

- Expand a power into a product.
- Write a product as a power.
- Simplify numeric and algebraic expressions containing integer exponents.
- Develop rules to simplify a product of powers, a power of a power, and a quotient of powers.
- Apply the properties of integer exponents to create equivalent expressions.

NEW KEY TERMS

- power
- base
- exponent

You have learned how to evaluate numeric expressions involving whole-number exponents.

In this lesson, you will develop the properties of integer exponents to generate equivalent numeric and algebraic expressions.

How can you use the properties of integer exponents to generate equivalent numeric expressions?

Sample answer:

I can rewrite powers in different ways.

- To multiply powers with the same base, I keep the base and add the exponents.
- To rewrite a power of a power, I keep the base and multiply the exponents.
- To divide powers with the same base, I keep the base and subtract the exponents.

MODULE 4 • TOPIC 1 • LESSON 1 511

Chunking the Activity

- Read and discuss the situation.
- Group students to complete the activity.
- Share and summarize.

Student Look-Fors

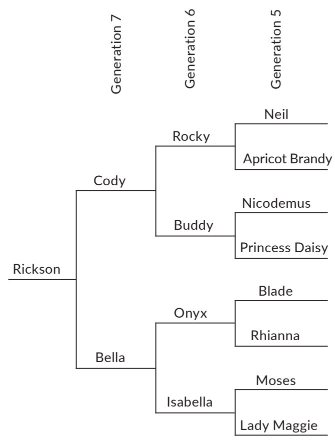
Whether students are demonstrating proficiencies related to TEKS A.1F:

- Can students identify and explain mathematical relationships?

Getting Started

Three Generations

Lucas adopted an eighth-generation purebred English Mastiff puppy that he named Rickson. The breeder provided documentation that verified a portion of Rickson's lineage going back three generations, as shown.



Ask Yourself . . .
What patterns do you notice?

A dog's lineage is similar to a person's family tree. It shows a dog's parents, grandparents, and great-grandparents.

1. How many parents does Rickson have? What are his parents' names?

   2 parents; Cody and Bella

2. How many grandparents does Rickson have?

   4 grandparents

3. How many great-grandparents does Rickson have?

   8 great-grandparents

4. What pattern is there in the number of dogs in each generation?

   The number of dogs in each generation is double the number of dogs in the previous generation.

EB STUDENT TIP

For all proficiency levels

Connect the words *base*, *exponent*, and *power* to a scenario such as a teacher starting a robotics team. The teacher recruits 3 students in the first round and, in the second round, asks these students to each recruit 3 more, for a total of 9. Continue at least once more to reach 27. As you narrate, consistently connect the word *base* to the original 3 students and the word *exponent* with the rounds of expansion.

Beginning: Have students enact the scenario by physically grouping up or by using manipulatives. Write the equation, prompting students to say or hold

(continued on next page)

512 MODULE 4 • TOPIC 1 • LESSON 1

ACTIVITY **1.1**

Review of Powers and Exponents

Lucas wants to trace Rickson's lineage back through all eight generations. How many sires (male parents) and dams (female parents) are there in each generation of Rickson's lineage?

1. Complete the second column of the table, Number of Sires and Dams, to show the total number of dogs in each generation.

Generation	Number of Sires and Dams	Expanded Notation	Power
Generation 7	2	2	2^1
Generation 6	4	2·2	2^2
Generation 5	8	2·2·2	2^3
Generation 4	16	2·2·2·2	2^4
Generation 3	32	2·2·2·2·2	2^5
Generation 2	64	2·2·2·2·2·2	2^6
Generation 1	128	2·2·2·2·2·2·2	2^7

Ask Yourself . . .
Why is the eighth generation not included as part of the table? Which dog is included in the eighth generation?

An expression used to represent the product of a repeated multiplication is a *power*. A **power** has a *base* and an *exponent*. The **base** of a power is the expression that is used as a factor in the repeated multiplication. The **exponent** of a power is the number of times that the base is used as a factor in the repeated multiplication.

$\text{base}^{\text{exponent}}$ ← **power**

WORKED EXAMPLE

You can write a power as a product by writing out the repeated multiplication.

$$2^7 = (2)(2)(2)(2)(2)(2)(2)$$

The power 2^7 can be read as:

- "two to the seventh power."
- "the seventh power of two."
- "two raised to the seventh power."

Chunking the Activity

- Read and discuss the situation.
- Group students to complete Question 1.
- Check in and share.
- Group students to complete Questions 2 and 3.
- Check in and share.
- Group students to complete Questions 4 and 5.
- Share and summarize.

STAMP THE LEARNING

The definitions and the Worked Example provide an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

EB STUDENT TIP *(continued)*

up a card of the corresponding word (*base*, *exponent*, or *power*) as you point to each quantity.

Intermediate: Use the sentence frame "The number of times I multiply the base ____ by itself is _____ times, which is the exponent. When I write this out mathematically as _________, this is a power."

Advanced/Advanced High: Have students use precise mathematical language to explain situations such as population growth or the spread of the common cold and why understanding exponential growth is essential in science, medicine, and many fields of study.

Think About . . .
How can I write the number of dogs in each generation as a repeated multiplication?

2. Label the third column *Expanded Notation*. Then, write each generation total as a product.

   See table on previous page.

3. Label the fourth column of the table *Power*. Then, write each generation total as a power.

   See table on previous page.

Ask Yourself . . .
How many generations are there before this dog in its lineage?

4. Suppose another dog is a thirteenth-generation purebred. How many dogs are in the first generation? Write your answer as a power and then use a calculator to determine the total number of dogs.

   2^{12} dogs $=$ 4096 dogs

5. How many total sires and dams are there in all three generations shown in Rickson's lineage? Explain your calculation.

   There are a total of 14 sires and dams in the three generations shown in Rickson's lineage.

   $8 + 4 + 2 = 14$

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ACTIVITY 1.2 Practice with Powers

In this activity, you will investigate the role of parentheses in expressions containing exponents.

1. Identify the base(s) and exponent(s) in each. Then, write each power as a product. Finally, evaluate the power.

   a. 5^3

   The base is 5, and the exponent is 3; $(5)(5)(5) = 125$

   b. $(-9)^5$

   The base is -9, and the exponent is 5; $(-9)(-9)(-9)(-9)(-9) = -59{,}049$

   c. -11^3

   The base is 11, and the exponent is 3. There is a factor of -1. $(-1)(11)(11)(11) = -1331$

   d. $(4)^5(3)^6$

   One base is 4, and its exponent is 5. The other base is 3, and its exponent is 6; $(4)(4)(4)(4)(4)(3)(3)(3)(3)(3)(3) = 746{,}496$

Think About . . .
When the negative sign is not in parentheses, it's not part of the base. Instead, there is a factor of -1.

2. Write each as a product. Then, calculate the product.

   a. -1^2

   $(-1)(1)(1) = -1$

   b. -1^3

   $(-1)(1)(1)(1) = -1$

   c. -1^4

   $(-1)(1)(1)(1)(1) = -1$

   d. -1^5

   $(-1)(1)(1)(1)(1)(1) = -1$

   e. $(-1)^2$

   $(-1)(-1) = 1$

   f. $(-1)^3$

   $(-1)(-1)(-1) = -1$

   g. $(-1)^4$

   $(-1)(-1)(-1)(-1) = 1$

   h. $(-1)^5$

   $(-1)(-1)(-1)(-1)(-1) = -1$

3. Consider Questions 2(f) and 2(h). What conclusion can you draw about a negative number raised to an odd power?

   A negative number raised to an odd power results in a negative product.

4. Consider Questions 2(e) and 2(g). What conclusion can you draw about a negative number raised to an even power?

   A negative number raised to an even power results in a positive product.

Chunking the Activity

- Read and discuss the situation.
- Group students to complete Questions 1 and 2.
- Check in and share.
- Complete Questions 3 and 4 as a class.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

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Chunking the Activity

- Read the Essential Question and activity summaries from Session 1.
- Read and discuss the directions and read the Worked Example.
- Group students to complete Question 1.
- Check in and share.
- Analyze the Worked Example.
- Group students to complete Questions 2–5.
- Share and summarize.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

ACTIVITY 1.3

Multiplying and Dividing Powers

1 gigabyte = 1024 megabytes

1 megabyte = 1024 kilobytes

1 kilobyte = 1024 bytes

File sizes of eBooks, podcasts, and song downloads depend on the complexity of the content and the number of images.

WORKED EXAMPLE

Suppose that a medium-sized eBook contains about 1 megabyte (MB) of information.

Since 1 megabyte is 1024 kilobytes (kB), and 1 kilobyte is 1024 bytes (B), you can multiply to determine the number of bytes in the eBook:

$$1 \text{ MB} = (1024 \text{ kB})\left(\frac{1024 \text{ B}}{1 \text{ kB}}\right) = 1{,}048{,}576 \text{ B}$$

There are 1,048,576 bytes in the eBook.

Remember . . .
Be sure to use units in your calculations.

1. One model of an eBook can store up to 256 MB of data. A USB jump drive can hold 2 GB of storage. Use the method shown in the Worked Example to calculate each.

   a. Calculate the number of bytes the eBook can store.

   $$256 \text{ MB} \cdot \frac{1024 \text{ kB}}{\text{MB}} \cdot \frac{1024 \text{ bytes}}{\text{kB}} = 268{,}435{,}456 \text{ bytes}$$

   b. A USB jump drive can hold 2 GB of storage. How many bytes can the USB jump drive hold?

   $$2 \text{ GB} \cdot \frac{1024 \text{ MB}}{\text{GB}} \cdot \frac{1024 \text{ kB}}{\text{MB}} \cdot \frac{1024 \text{ bytes}}{\text{kB}} = 2{,}147{,}483{,}648 \text{ bytes}$$

   c. How many times more storage space does the jump drive have than the eBook? Show your work.

   $$\frac{(2 \cdot 1024 \cdot 1024 \cdot 1024)}{(256 \cdot 1024 \cdot 1024)} = \frac{(2 \cdot 1024)}{256} = \frac{2048}{256} = 8$$

   The jump drive has 8 times the amount of storage as the eBook.

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WORKED EXAMPLE

Computers use binary math, or the base-2 system, instead of the base-10 system.

Base 10	Base 2
$10^1 = 10$	$2^1 = 2$
$10^2 = (10)(10) = 100$	$2^2 = (2)(2) = 4$
$10^3 = (10)(10)(10) = 1000$	$2^3 = (2)(2)(2) = 8$

2. Revisit Question 1, parts (a) through (c), by rewriting each factor and either your product or quotient as a power of 2.

   a. $2^8 \cdot 2^{10} \cdot 2^{10} = 2^{28}$

   b. $2^1 \cdot 2^{10} \cdot 2^{10} \cdot 2^{10} = 2^{31}$

   c. $\frac{2^{31}}{2^{28}} = 2^3$

3. Analyze your answers to Question 2. What do you notice about all the bases in Question 2?

   All the bases are the same, which is 2.

4. In parts (a) and (b), how does the exponent in each product relate to the exponents in the factors?

   The exponent in each product is the sum of the exponents of the factors.

5. In part (c), how does the exponent in the quotient relate to the exponents in the numerator and denominator?

   The exponent in the quotient is the difference of the exponents in the numerator and the exponents in the denominator.

Optimizing Learning

Questions 3–5 provide options that support students to maximize transfer and generalization.

STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

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Chunking the Activity

- Group students to complete Questions 1–5.
- Check in and share.
- Analyze the Worked Example.
- Group students to complete Questions 6–8.
- Check in and share.
- Group students to complete Questions 9–11.
- Share and summarize.
- Return to the lesson opener and read the Essential Question.

ACTIVITY 1.4 Product of Powers

In this activity, you will explore different expressions to develop rules to evaluate powers.

1. Rewrite each expression as a product using expanded notation. Then, identify the base or bases and record the number of times the base is used as a factor.

   a. $2^4 \cdot 2^3$

   (2)(2)(2)(2)(2)(2)(2); base is 2; the base is used as a factor 7 times.

   b. $(-3)^3(-3)^3$

   (−3)(−3)(−3)(−3)(−3)(−3); base is −3; the base is used as a factor 6 times.

   c. $(4)(4^5)$

   (4)(4)(4)(4)(4)(4); base is 4; the base is used as a factor 6 times.

   d. $(5^2)(6^2)(5^3)(6)$

   (5)(5)(6)(6)(5)(5)(5)(6); bases are 5 and 6; the base 5 is used as a factor 5 times; the base 6 is used as a factor 3 times.

   e. $(9^3)(4^2)(9^2)(4^5)$

   (9)(9)(9)(4)(4)(9)(9)(4)(4)(4)(4)(4); bases are 9 and 4; the base 9 is used as a factor 5 times; the base 4 is used as a factor 7 times.

2. Rewrite each of your answers from Question 1 as a power or a product of powers.

   a. 2^7

   b. $(-3)^6$

   c. 4^6

   d. $5^5 \cdot 6^3$

   e. $9^5 \cdot 4^7$

SELF-MONITORING STRATEGY

Look for students who are working to build relationships within the group to improve teamwork. If groups are struggling to work together effectively, consider facilitating a team-building activity.

Refer to the Course and Implementation Guide for further details on these look-fors.

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3. What relationship do you notice between the exponents in the original expression and the number of factors?

The number of factors is the sum of the exponents with the same base.

4. Write a rule that you can use to multiply powers.

To multiply powers with the same base, I must keep the base the same and then add the exponents.

5. Use your rule to write an equivalent expression as a power or product of powers in simplest form.

a. $5^2 \cdot 5^3$

5^5

b. $x^4 \cdot x^6$

x^{10}

c. $(-3)^3(-3)^5$

$(-3)^8$

d. $(-a)^2(-a)$

$(-a)^3$

e. $(6)(4^7)(6^8)$

$(4^7)(6^9)$

f. $(c^3)(d^9)(c^{12})(d^4)$

$c^{15}d^{13}$

Student Look-Fors

Whether students are demonstrating proficiencies related to TEKS A.1G:

- Do students defend their mathematical reasoning?
- Do students use precise mathematical language when communicating?

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STAMP THE LEARNING

The Worked Example provides an opportunity for explicit instruction. Interact with this information as a class and encourage students to restate or explain the information in their own words.

Questions 6–8 present an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice with the power of a power rule, assign Skills Practice Set A for this lesson.

Optimizing Learning

Questions 6 and 7 provide options that support students to highlight patterns, critical features, big ideas, and relationships

A power can also be raised to a power.

WORKED EXAMPLE

The exponential expression $(4^2)^3$ is a power of a power. It can be written as two repeated multiplication expressions using the definition of a power.

$(4^2)^3 = (4^2)(4^2)(4^2)$

$= (4 \cdot 4) \cdot (4 \cdot 4) \cdot (4 \cdot 4)$

There are 6 factors of 4.

6. Use the definition of a power to write repeated multiplication expressions for each power of a power, as modeled in the Worked Example. Then, record the number of factors.

a. $(8^2)^3$
$(8 \cdot 8) \cdot (8 \cdot 8) \cdot (8 \cdot 8)$;
number of factors = 6

b. $(5^4)^2$
$(5 \cdot 5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5 \cdot 5)$;
number of factors = 8

c. $-(6^1)^6$
$-(6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6)$;
number of factors = 6

d. $((-6)^2)^2$
$(-6 \cdot (-6)) \cdot (-6 \cdot (-6))$;
number of factors = 4

7. What relationship do you notice between the exponents in each expression in Question 5 and the number of factors? Write each expression as a single power.

The number of factors is the product of the exponents in each expression.

8^6, 5^8, $-(6)^6$, $(-6)^4$

8. Write a rule that you can use to determine a power of a power.

To determine a power of a power, keep the base and multiply the exponents.

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9. Write an equivalent expression in simplest form using the rules that you wrote.

a. $6^4 \cdot 6^3$

6^7

b. $y^7 \cdot y^8$

y^{15}

c. $(4^3)^5$

4^{15}

d. $(w^7)^3$

w^{21}

e. $r^5 \cdot r^2 \cdot r$

r^8

f. $((2)(3))^4$

6^4

10. Mason says that $2^6 = 12$. Luna says that $2^6 = 64$. Who is correct? Explain your reasoning.

Luna is correct. Mason multiplied 2 by 6 instead of raising 2 to the sixth power, which is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ or 64.

11. Nahimana says that $2^2 + 2^3 = 2^5$, and Sebastian says that $2^2 + 2^3 \neq 2^5$. Who is correct? Explain your reasoning.

Sebastian is correct. $2^2 \cdot 2^3 = 2^5$.

MODULE 4 • TOPIC 1 • LESSON 1 521

Chunking the Activity

- Read the Essential Question and activity summaries from Sessions 1 and 2.
- Group students to complete Questions 1–3.
- Check in and share.
- Have students individually complete Questions 4.
- Share and summarize.

ACTIVITY 1.5 Quotient of Powers

Now, let's investigate what happens when you divide powers with like bases.

1. Write each numerator and denominator as a product. Then, write an equivalent expression in simplest form using exponents.

a. $\frac{9^5}{9^2}$

$\frac{(9 \cdot 9 \cdot 9 \cdot 9 \cdot 9)}{(9 \cdot 9)}$; 9^3

b. $\frac{5^6}{5^3}$

$\frac{(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)}{(5 \cdot 5 \cdot 5)}$; 5^3

c. $\frac{x^8}{x^6}$

$\frac{(x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x)}{(x \cdot x \cdot x \cdot x \cdot x \cdot x)}$; x^2

d. $\frac{10^2}{10}$

$\frac{(10 \cdot 10)}{10}$; 10^1

2. What relationship do you notice between the exponents in the numerator and denominator and the exponents in the simplified expression?

When the bases are the same, an exponent in the simplified expression is the difference of the exponents in the numerator and denominator.

3. Write a rule that you can use to divide with powers.

To simplify a quotient of two powers, keep the base and subtract the exponent in the denominator from the exponent in the numerator.

4. Write an equivalent expression in simplest form using the rule that you wrote for a quotient of powers.

a. $\frac{6^8}{6^3}$

6^5

b. $-\frac{t^7}{t^5}$

$-t^2$

c. $\frac{2^3}{3^2}$

Bases are not the same, so it cannot be simplified using exponents.

$\frac{2^3}{3^2} = \frac{8}{9}$

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ACTIVITY 1.6

Powers Equal to 1 and Numbers Less Than 1

You know that any number divided by itself is 1. How can you use that knowledge to develop another rule to evaluate powers?

Consider each representation of 1.

$\frac{4}{4} = 1$ $\quad\quad$ $\frac{9}{9} = 1$ $\quad\quad$ $\frac{25}{25} = 1$ $\quad\quad$ $\frac{x}{x} = 1$

1. Rewrite the numerator and denominator of each fraction as a power. Do not simplify.

   $\frac{4^1}{4^1}$ or $\frac{2^2}{2^2}$ $\quad\quad$ $\frac{9^1}{9^1}$ or $\frac{3^2}{3^2}$ $\quad\quad$ $\frac{25^1}{25^1}$ or $\frac{5^2}{5^2}$ $\quad\quad$ $\frac{x^1}{x^1}$

2. Next, simplify the fractions you just wrote using the quotient of powers rule. Leave your answer as a power. What do you notice?

   4^0 or 2^0 $\quad\quad$ 9^0 or 3^0 $\quad\quad$ 25^0 or 5^0 $\quad\quad$ x^0

3. Write a rule that you can use when raising any base to the zero power.

   If a base is raised to the zero power, then the result is 1.

Let's determine how to use powers to represent numbers that are less than 1.

4. Let's start with 1 and multiply by 10 three times.

   a. Complete the representation. Write each as a power.

	1	=	10^0
Multiply by 10 =	10	=	10^1
Multiply by 10 =	$10 \cdot 10$	=	10^2
Multiply by 10 =	$10 \cdot 10 \cdot 10$	=	10^3

An exception is that 0^0 is not equal to 1, because that would mean that using zero as a factor zero times would give you 1, and that's not possible.

You know that you can use powers to represent numbers that are greater than or equal to 1.

Chunking the Activity

- Group students to complete Questions 1–3.
- Check in and share.
- Complete Questions 4 and 5 as a class.
- Check in and share.
- Group students to complete Question 6.
- Check in and share.
- Group students to complete Questions 7–12.
- Share and summarize.

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b. Describe what happens to the exponents as the number becomes greater.

The exponents increase as the number becomes greater. The exponents are positive integers.

5. Now, let's start with 1 and divide by 10 three times.

a. Complete the representation. Write the division as a fraction and then rewrite using the definition of powers. Next, apply the quotient of powers rule, and finally, simplify each expression.

$$1 = \frac{10^0}{10^0} = 10^{0-0} = 10^0$$

Divide by 10 = $\frac{1}{10} = \frac{10^0}{10^1} = 10^{0-1} = 10^{-1}$

Divide by 10 = $\frac{1}{100} = \frac{10^0}{10^2} = 10^{0-2} = 10^{-2}$

Divide by 10 = $\frac{1}{1000} = \frac{10^0}{10^3} = 10^{0-3} = 10^{-3}$

b. Describe what happens to the exponents as the number becomes less.

The exponents decrease as the number becomes smaller. The exponents are negative integers.

c. Write each of the powers as a decimal.

$10^0 = 1.0$

$10^{-1} = 0.1$

$10^{-2} = 0.01$

$10^{-3} = 0.001$

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6. Rewrite each sequence of numbers using the definition of powers.

a. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8$

$2^{-3}, 2^{-2}, 2^{-1}, 2^0, 2^1, 2^2, 2^3$

b. $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27$

$3^{-3}, 3^{-2}, 3^{-1}, 3^0, 3^1, 3^2, 3^3$

c. Describe the exponents in the sequence.

As the numbers become greater, the exponents become greater. The exponents are integers from −3 to 3.

7. Write an equivalent expression in simplest form using the quotient of powers rule. Then, write each as a decimal.

a. $\frac{10^0}{10^3}$

10^{-3}; 0.001

b. $\frac{10^0}{10^5}$

10^{-5}; 0.00001

c. $\frac{10^0}{10^4}$

10^{-4}; 0.0001

8. Complete the table shown.

Unit	Number of Grams	Number of Grams as an Expression with a Positive Exponent	Number of Grams as an Expression with a Negative Exponent
Milligram	$\frac{1}{1000}$	$\frac{1}{10^3}$	10^{-3}
Microgram	$\frac{1}{1,000,000}$	$\frac{1}{10^6}$	10^{-6}
Nanogram	$\frac{1}{1,000,000,000}$	$\frac{1}{10^9}$	10^{-9}
Picogram	$\frac{1}{1,000,000,000,000}$	$\frac{1}{10^{12}}$	10^{-12}

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9. Write an equivalent expression in simplest form such that the exponent is positive.

a. 8^{-4}

$\frac{1}{8^4}$

b. 5^{-6}

$\frac{1}{5^6}$

c. p^{-5}

$\frac{1}{p^5}$

d. $(4^{-2})(3^{-3})$

$\frac{1}{4^2} \cdot \frac{1}{3^3}$

10. Complete the table.

Given Expression	Expression with a Positive Exponent	Value of Expression
$\frac{1}{3^{-2}}$	3^2	9
$\frac{1}{4^{-2}}$	4^2	16
$\frac{1}{5^{-2}}$	5^2	25
$\frac{2^{-2}}{1}$	$\frac{1}{2^2}$	$\frac{1}{4}$
$\frac{3^{-2}}{1}$	$\frac{1}{3^2}$	$\frac{1}{9}$
$\frac{5^{-2}}{1}$	$\frac{1}{5^2}$	$\frac{1}{25}$

11. Describe how to rewrite any expression with a negative exponent in the numerator.

Write the reciprocal of the fraction with the power with its opposite (positive) exponent in the denominator.

12. Describe how to rewrite any expression with a negative exponent in the denominator.

Write the reciprocal of the fraction with the power with its opposite (or now positive) exponent.

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Talk the Talk

Simplifying

In this lesson, you have developed rules for operating with powers. A summary of these rules is shown in the table.

Properties of Powers	Words	Rule
Product rule of powers	To multiply powers with the same base, keep the base and add the exponents.	$a^m a^n = a^{m+n}$
Power of a power rule	To simplify a power of a power, keep the base and multiply the exponents.	$(a^m)^n = a^{mn}$
Quotient of powers rule	To divide powers with the same base, keep the base and subtract the exponents.	$\frac{a^m}{a^n} = a^{m-n}$, if $a \neq 0$
Zero power	The zero power of any number, except for 0, is 1.	$a^0 = 1$, if $a \neq 0$
Negative exponents in the numerator	An expression with a negative exponent in the numerator and a 1 in the denominator equals 1 divided by the power with its opposite exponent placed in the denominator.	$a^{-m} = \frac{1}{a^m}$, if $a \neq 0$ and $m > 0$
Negative exponents in the denominator	An expression with a negative exponent in the denominator and a 1 in the numerator equals the power with its opposite exponent.	$\frac{1}{a^{-m}} = a^m$, if $a \neq 0$ and $m > 0$

Chunking the Activity

- Discuss in groups.
- Group students to complete the activity.
- Share and summarize.
- Have students answer the Essential Question on the lesson opener.

This Activity presents an opportunity to assess students' understanding of the essential content of the lesson. Use student responses to determine when to schedule Learning Individually Days. To provide additional practice on rewriting expressions using properties of powers, assign Skills Practice Sets B and C for this lesson.

Write an equivalent expression in simplest form using the properties of powers.

1. $2a^8 \cdot 2a^6$

   $4a^{14}$

2. $4b^2 \cdot 8b^9$

   $32b^{11}$

3. $-3c \cdot 5c^3 \cdot 2c^9$

   $-30c^{13}$

4. $(3d^2)^3$

   $27d^6$

5. $(10ef^3)^5$

   $100{,}000e^5f^{15}$

6. $\frac{f^8}{f^3}$

   f^5

7. $\frac{10g^4}{5g^6}$

   $\frac{2}{g^2}$

8. $\frac{30h^8}{15h^2}$

   $2h^6$

9. $\frac{35i^7j^3}{7i^2j^3}$

   $5i^5$

10. $\left(\frac{a^2}{a^5}\right)^0$

    1 (assuming $a \neq 0$)

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11. $\frac{2^2}{2^6}$

$\frac{1}{2^4} = \frac{1}{16}$

12. $(4x^2)(3x^5)$

$12x^7$

13. $(9^4)(9^{-5})$

$\frac{1}{9}$

14. $(8^0)(8^{-2})$

$\frac{1}{8^2} = \frac{1}{64}$

15. $\frac{3^{-3}}{3^{-3}}$

1

16. $\frac{4^{-2}}{4^{-3}}$

4

17. $\frac{(-3)^2}{(-3)^4}$

$\frac{1}{9}$

18. $\frac{h^3}{h^5}$

$\frac{1}{h^2}$

19. $\frac{x^4}{x^5}$

$\frac{1}{x^1} = \frac{1}{x}$

20. $\frac{m^2p^{-2}}{m^4p^3}$

$\frac{1}{m^2p^5}$

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Lesson 1 Assignment

Write

Use the term *base*, *power*, or *exponent* to complete each sentence.

1. The exponent of a power is the number of times that the factor is repeatedly multiplied.
2. An expression used to represent a factor as repeated multiplication is called a power.
3. The base of a power is the repeated factor in a power.

Remember

Properties of Powers	Words	Rule
Product of powers rule	To multiply powers with the same base, keep the base and add the exponents.	$a^m a^n = a^{m+n}$
Power of a power rule	To simplify a power of a power, keep the base and multiply the exponents.	$(a^m)^n = a^{mn}$
Quotient of powers rule	To divide powers with the same base, keep the base and subtract the exponents.	$\frac{a^m}{a^n} = a^{m-n}$, if $a \neq 0$

Practice

1. As the principal of a middle school, Mr. Park is in charge of notifying his staff about school delays or cancellations due to weather, power outages, or other unexpected events. Mr. Park starts a phone chain by calling three staff members. Each of these staff members then calls three more staff members, who each call three more staff members. This process completes the calling list.

   a. Excluding Mr. Park, how many staff members are there at the middle school? Explain your calculation.

   There are 39 staff members at the middle school.
   $3 + 3(3) + 3(3)(3) = 3 + 9 + 27 = 39$

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Lesson 1 Assignment

b. Complete the first three columns in the table.

	Round 1	Round 2	Round 3	Round 4
Number of calls made	3	9	27	81
Expanded notation	3	$3 \cdot 3$	$3 \cdot 3 \cdot 3$	$3 \cdot 3 \cdot 3 \cdot 3$
Power	3	3^2	3^3	3^4

c. Dr. Martinez, superintendent of the school district, decides that she should start the phone chain instead of the school principals. She starts the calling list by calling each of the principals of the three schools in her district. The principals continue the phone chain as described previously. Explain why the table in part (b) can be used to represent Dr. Martinez's phone chain.

The table in part (b) can be used to represent Dr. Martinez's phone chain because each staff member is still calling three other staff members. The table would just require another column to show a fourth round of calls.

d. Complete the fourth column in the table to represent the fourth round of calls in Dr. Martinez's phone chain.

See table in 1b.

e. How many calls are made in the fourth round?

There are 81 calls made in the fourth round.

f. Excluding Dr. Martinez, how many principals and staff members are there in the entire school district? Explain your calculation.

There are 120 principals and staff members in the entire school district.

$3 + 3(3) + 3(3)(3) + 3(3)(3)(3) = 3 + 9 + 27 + 81 = 120$

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Lesson 1 Assignment

2. The *hertz* (Hz) is a unit of frequency that represents the number of complete cycles per second. It is used to measure repeating events, both scientific and general. For instance, a clock ticks at 1 Hz. One scientific application is the electromagnetic spectrum, or the range of all possible frequencies of electromagnetic waves. The spectrum includes frequencies from everyday contexts, such as radio and TV signals, microwaves, light (infrared, visible, and ultraviolet), and X-rays.

Name	Frequency
1 kilohertz (kHz)	1000 Hz
1 megahertz (MHz)	1000 kHz
1 gigahertz (GHz)	1000 MHz
1 terahertz (THz)	1000 GHz

For each question, use powers to write a mathematical expression. Then, evaluate each expression. Express your answer as a power.

a. How many hertz are in 1 gigahertz? 1 terahertz?
10^9 Hz; 10^{12} Hz

b. A television channel has a frequency of 60 megahertz. What is the channel's frequency in hertz?
The channel's frequency is 60 million hertz.

c. The frequency of a microwave is 30 gigahertz. What is the microwave's frequency in hertz?
The microwave's frequency is 30 billion hertz.

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d. The frequency of a visible ray of light is 1000 terahertz. A radio station transmits at a frequency of 100 megahertz. How many times greater is the frequency of the light than the frequency of the radio station?

The frequency of the light is 10 million times greater than the frequency of the radio station.

3. Each expression has been simplified incorrectly. Explain the mistake that occurred and then make the correction.

a. $(-2x)^3 = 8x^3$

The negative for the power was not raised to the third power.

$(-2x)^3 = -8x^3$

b. $\frac{16x^5}{4x} = 12x^4$

The numbers 16 and 4 were subtracted. The fraction $\frac{16}{4}$ should be simplified instead. $\frac{16x^5}{4x} = 4x^4$

c. $(x^2y^4)^3 = x^6y^7$

The exponents of base y were added instead of multiplied.
$(x^2y^4)^3 = x^6y^{12}$

d. $(x^5y^7)(x^2yz) = x^7y^7z$

The y in the second expression has an exponent of 1; it was missed. $(x^5y^7)(x^2yz) = x^7y^8z$

4. When you take a picture, the camera shutter controls how much light reaches the film or the digital image sensor. The shutter speed is the amount of time, in seconds, that the shutter stays open. Write each shutter speed as a power with a negative exponent.

a. $\frac{1}{4}$ second

2^{-2} second

b. $\frac{1}{8}$ second

2^{-3} second

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5. True or False: A number raised to a negative power is always a negative number. Give an example to support your answer.

   False; $2^{-2} = \frac{1}{4}$

6. Give an example of a number raised to a negative exponent that is a negative number.

   Answers will vary.

   A negative base raised to an odd negative exponent is a negative number.

7. Write an equivalent expression in simplest form using the properties of powers. Show your work.

   a. $\frac{10^2}{10^5}$

   $\frac{1}{10^3} = \frac{1}{1000}$

   b. $\frac{3^{-5}}{3^{-5}}$

   1

   c. $(6x^4)(2x^{-2})$

   $12x^2$

   d. $(7^{-6})(7^4)$

   $\frac{1}{7^2} = \frac{1}{49}$

   e. $(4^0)(4^{-3})$

   $\frac{1}{4^3} = \frac{1}{64}$

   f. $\frac{5^2}{5^{-2}}$

   $5^4 = 625$

   g. $\frac{4^{-1}}{4^2}$

   $\frac{1}{4^3} = \frac{1}{64}$

   h. $\frac{15p^4}{3p^9}$

   $\frac{5}{p^5}$

   i. $\frac{m^{-2}}{m^{-6}}$

   m^4

   j. $\frac{q^{-2}r^{-3}}{q^6r^{-4}}$

   $\frac{r}{q^8}$

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Prepare

Simplify each expression.

1. $\frac{1}{3^2}$

   $\frac{1}{9}$

2. $-(2^5)^2$

   $-2^{10} = -1024$

3. $\frac{2}{5} \cdot 10$

   4

4. $\frac{32 + 8}{80 \div 2}$

   1

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